Rossmoyne Senior High School

MATHEMATICS APPLICATIONS UNITS 1 AND 2
Section One:
Calculator-free

SOLUTIONS

Student Number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
          correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1.   The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer Booklet.

3.   You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4.   Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   * Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   * Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5.   **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6.   It is recommended that you **do not use pencil**, except in diagrams.

7.   The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section One: Calculator-free **(52 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(8 marks)**

(a)   If $A = 2\sqrt{\dfrac{hw}{10}}$, determine A when $h = 4$ and $w = 40$. (2 marks)

$$A = 2 \times \sqrt{\dfrac{4 \times 40}{10}}$$
$$= 2 \times \sqrt{16}$$
$$= 8$$

(b)   Solve the following equations algebraically, showing working.

(i) $3x - 2 = 18 - x$ (2 marks)

$$4x = 20$$
$$x = 5$$

(ii) $\dfrac{2}{3}(x - 1) = -2x + 10$ (2 marks)

$$2x - 2 = -6x + 30$$
$$8x = 32 \implies x = 4$$

(iii) $\dfrac{x}{2} + \dfrac{1}{2} = \dfrac{2x}{5}$ (2 marks)

$$5x + 5 = 4x$$
$$x = -5$$

See next page

Question 2   **(6 marks)**

A set of test scores were 12, 9, 4, 16, 13, 2, 10, 11, 10, 15.

(a)   State the range of the scores.   (1 mark)

$$16 - 2 = 14$$

(b)   The minimum and maximum scores are 2 and 16 respectively. Calculate the other
        statistics that would be required to construct a boxplot for these scores.   (3 marks)

2, 4, **9**, 10, 10   (M)   11, 12, **13**, 15, 16

Median is 10.5

Lower Quartile is 9

Upper Quartile is 13

(c)   Use a calculation to decide whether or not the set of scores contains an outlier.   (2 marks)

IQR = 13 - 9 = 4

LQ - 1.5×IQR = 9 - 1.5×4 = 3

Any score below 3 is an outlier, so the
score of 2 is an outlier - the set of
scores **does** contain an outlier.

See next page

Question 3 **(6 marks)**

Consider the matrices $A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

(a) If possible, calculate the following. If not possible, explain why.

(i) $3B$ (1 mark)

$$3\begin{bmatrix} -2 & 5 \end{bmatrix} = \begin{bmatrix} -6 & 15 \end{bmatrix}$$

(ii) $A - D$ (1 mark)

$$\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

(iii) DB (1 mark)

Not possible, as number of columns in D not equal to number of rows in B.

(iv) AC (1 mark)

$$\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

(b) If $\begin{bmatrix} p & 1 \end{bmatrix} \times D = \begin{bmatrix} q & -6 \end{bmatrix}$, determine the values of p and q. (2 marks)

$$\begin{bmatrix} p & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} q & -6 \end{bmatrix}$$

$$p + 2 = q$$

$$-2p = -6 \implies p = 3$$

$$q = 5$$

See next page

Question 4   **(7 marks)**

(a) To calculate the acceleration, a, of a body that changes speed from u to v over a distance of s, the formula $v^2 = u^2 + 2as$ can be used.

Determine a when $u = 2$, $v = 6$ and $s = 8$.   (3 marks)

$$6^2 = 2^2 + 2 \times a \times 8$$
$$36 = 4 + 16a$$
$$32 = 16a$$
$$a = 2$$

(b) The table below shows the volume, in cm³, of cylinders of different whole number heights (cm) and radii (cm), calculated using the formula $V = 3r^2h$.

$V = 3r^2h$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
$h = 1$	3	12	27	48	75
$h = 2$	6	24	54	96	150
$h = 3$	9	**A**	81	144	225
$h = 4$	12	48	108	192	**B**

(i) Determine the values of A and B in the table.   (2 marks)

$$A = 3 \times 2^2 \times 3 = 36$$

$$B = 3 \times 5^2 \times 4 = 300$$

(ii) Some cylinders with different dimensions have exactly the same volume. For example, the cylinders with $h = 4$, $r = 2$ and $h = 1$, $r = 4$ both have the same volume of 48 cm³.

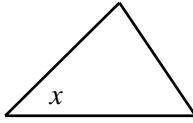Determine the whole number dimensions of two cylinders that both have a volume of 108 cm³.   (2 marks)

$$h = 4,\ r = 3 \text{ and } h = 1,\ r = 6$$

See next page

Question 5 **(9 marks)**

A scalene triangle is shown below, not to scale, with smallest angle x.

(a) Consider the case where the other two angles are 10° and 20° larger than x respectively.

(i) Explain why $x + x + 10 + x + 20 = 180$. (1 mark)

> The three angles are x, $x + 10$ and $x + 20$,
> and their sum must be 180°.

(ii) Solve the equation in (i) and hence state the sizes of the three angles in the triangle. (3 marks)

> $3x + 30 = 180$
> $3x = 150$
> $x = 50°$
>
> Angles are $50°$, $60°$ and $70°$

(b) Consider another case, where one of the larger angles is three times the size of x, and the largest angle is 60° more than twice x.

(i) Use this information to write down an equation. (2 marks)

> $x + 3x + 2x + 60 = 180$

(ii) Solve your equation from (i) and hence state the size of the largest angle in the triangle. (3 marks)

> $6x + 60 = 180$
> $6x = 120$
> $x = 20°$
>
> $3x = 60°$, $2x + 60 = 100°$
>
> Largest angle is $100°$

See next page
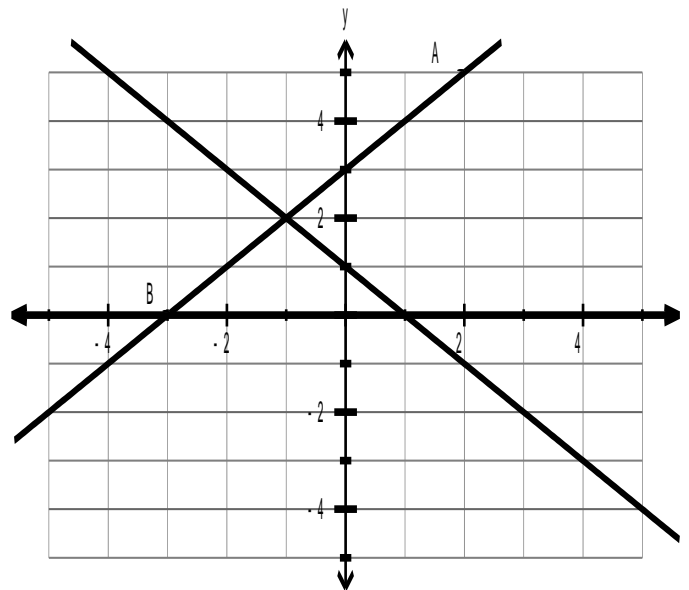
Question 6　　　　　　　　　　　　　　　　　　　　　　　**(8 marks)**

(a)　Two linear functions are given by $y = \dfrac{x}{2} + 4$ and $y = 1 - x$. The point A(2, 5) lies on one of the lines but not the other. Show this algebraically.　　　(2 marks)

> Substitute $x = 2$ and $y = 5$ into both equations to see which is true.
>
> $5 = \dfrac{2}{2} + 4 \Rightarrow 5 = 5 \therefore$ True, lies on line　　$5 = 1 - 2 \Rightarrow 5 = -1 \therefore$ False, does not lie on line

(b)　Plot the points A(2, 5) and B(-3, 0) on the axes below and then draw a straight line through them.　　　(1 mark)

(c)　Determine the rule for the linear function passing through points A and B.　　　(2 marks)

$$y = x + 3$$

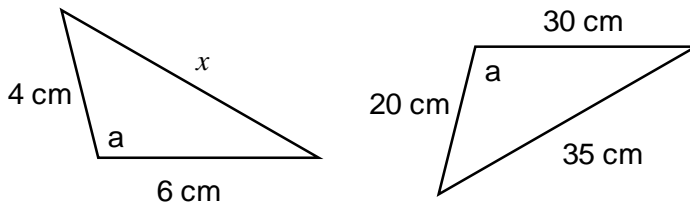(d)　Graph the function $y = 1 - x$ on the above set of axes.　　　(2 marks)

(e)　From the graph, state the coordinates of the point of intersection of the line passing through A and B and the line $y = 1 - x$.　　　(1 mark)

(-1, 2)

See next page

Question 7 **(8 marks)**

(a) The two triangles shown below, not to scale, are similar.

(i) Determine the length of x. (2 marks)

$$20 \div 4 = 5$$

$$35 \div 5 = 7 \text{ cm}$$

(ii) If the area of the smaller triangle is A cm², write down an expression for the area of the larger triangle. (2 marks)

$$A \times 5^2 = 25A$$

(b) A soft drink is retailed in cylindrical cans of radius 3 cm and height 11 cm. A promotional model of the can is constructed so that its shape is similar to the retail version but with a radius of 30 cm.

(i) What is the height of the promotional model? (1 mark)

$$30 \div 3 = 10$$

$$11 \times 10 = 110 \text{ cm}$$

(ii) If the retail version of the can holds 300 mL, what is the capacity, in litres, of the promotional model? (3 marks)

$$300 \text{ mL} = 0.3 \text{ L}$$

$$0.3 \times 10^3 = 300 \text{ L}$$

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____